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Quantum cosmology and AdS/CFT

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Abstract: In this paper we study the creation of brane-worlds in AdS bulk. We first consider the simplest case of onebranes in AdS_3. In this case we are able to properly describe the creation of a spherically symmetric brane-world deriving a general expression for its wavefunction. Then, we sketch the AdS_{d+1} set-up within the context of the WKB approximation. Finally, we comment on these scenarios in light of the AdS/CFT correspondence.

Keywords: Extra Large Dimensions, AdS/CFT Correspondance, Cosmology of Theories beyond the SM.

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1. Introduction

The idea that spacetime has more than four dimensions is actually quite old. Already in the 1920's, Kaluza suggested that gravity and electromagnetism can be interpreted as the degrees of freedom of the metric of a five-dimensional spacetime [1]. Later, Klein [2] gave an explanation for the fact that the extra dimension is not observed by suggesting that the extra dimension is compact and very small. Since then the idea has been studied from many different perspectives e.g. in Kaluza-Klein supergravity theories and also in string theories — where more than three spatial dimensions naturally arise but the extra dimensions are usually assumed to be of Planck size for not been directly observable. In another direction it has been suggested [3] that spacetime can have more than three *non-compact* spatial dimensions if we live on a four-dimensional domain wall which is embedded in the higher dimensions. More recently, there has been a renewed interest in the topic since progresses in string theories [4] have modified the old scenario (where the extra dimensions cannot exceed the tiny scale $\sim 1\,\mathrm{TeV}^{-1} \sim 10^{-19}\,\mathrm{m}$) suggesting that Standard Model gauge interactions could be confined to a four-dimensional subspace — or brane-world — whereas gravity can still propagate in the whole bulk spacetime. Actually, the possibility that part of the standard model particles live in large (TeV) extra dimensions was first put forward in connection to the problem of supersymmetry breaking in string theory [5]. These scenarios presents us with the enticing possibility to explain some long-standing particle physics problems by geometrical means [6, 7, 8].

In the canonical example of [6], spacetime is a direct product of ordinary four-dimensional spacetime and a (flat) spatial d-torus of common linear size r_c. Within this simple model, the large hierarchy between the weak scale and the fundamental scale of gravity can be eliminated. However, the hierarchy only arises in the presence of a large volume for the compactified dimension which is very difficult to justify. A more compelling scenario was introduced by Randall and Sundrum (herein RS). Reviving an old idea [3], RS proposed a set-up with the shape of a gravitational condenser in which two branes of opposite tension (which gravitationally repel each other) are stabilized by a slab of anti-de Sitter (AdS) space [7]. In this model the extra dimension is strongly curved, and the distance scales on the brane with negative tension are exponentially smaller than those on the positive tension brane. Such exponential suppression can then naturally explain why the observed physical scales are so much smaller than the Plank scale. In further work RS found that gravitons can be localized on a brane which separates two patches of AdS_5 spacetime [8], suggesting that it is possible to have an infinite extra dimension [9]. The question whether this scenario reproduces the usual four-dimensional gravity beyond the Newton's law has been analyzed [10] and cosmological considerations of models with large extra dimensions confirms that they are at least consistent candidates for describing our world [11]. These ideas have raised a lot of interest in the subject and several groups have begun to work on possible experimental signatures of the extra dimension(s) [12].

In this paper we shall discuss the creation of brane-worlds in AdS bulk. The approximation scheme to be used is the minisuperspace restriction of the canonical Wheeler-DeWitt formalism. The basic idea of this approach, commonly adopted in quantum cosmology calculations [13], is to separate the space-like metric into "modes", and then insist that all the "translational" modes are "frozen out" by using the classical field equations, leaving only the scale factor to be quantized. The outline of the paper is as follows. We begin in section 2 by deriving a brane-big-bang in AdS_3. This lower-dimensional model provides a simple setting in which certain basic physical phenomena can be easily demonstrated while avoiding the mathematical complexities associated with the higher-dimensional counterparts. In section 3 we consider multi-dimensional brane-worlds, discussing the possible cosmologies within the WKB approximation. In section 4 we analyze the implications of the AdS/CFT correspondence to quantum cosmology.

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2. Brane-world in AdS_3

2.1 Wheeler-DeWitt equation

In this section we consider the creation of a onebrane in AdS_3 within the framework of quantum cosmology [13]. Thus the universe will initially be described by three-dimensional anti-de Sitter space in which onebrane bubbles can nucleate spontaneously. As we shall see below, these bubbles appear (classically) at a critical size and then expand.

We thus begin by considering the action for a onebrane coupled to gravity,

$$S_{\rm tot} = \frac{L_p}{16\pi}\int_\Omega d^3x\sqrt{g}\left(R+\frac{2}{\ell^2}\right)+\frac{L_p}{8\pi}\int_{\partial\Omega} d^2x\sqrt{\gamma}\;\mathfrak{K}+T\int_{\partial\Omega} d^2x\sqrt{\gamma}\,, \tag{2.1}$$

where $\mathfrak{K}$ stands for the trace of the extrinsic curvature of the boundary, γ is the induced metric on the brane, and T is the brane tension.[1] The first term is the usual Einstein-Hilbert (EH) action with a negative cosmological constant ($\Lambda = -1/\ell^2$). The second term is the Gibbons-Hawking (GH) boundary term, necessary for a well-defined variational problem [15]. The third term corresponds to a constant "vacuum energy", i.e. a cosmological term on the boundary.

We wish to consider a brane which bounds two regions of AdS_3. If we further specialize to the case of spherical symmetry [14] where

$$ds_3^2 = -\left(1+\frac{y^2}{\ell^2}\right)dt^2+\left(1+\frac{y^2}{\ell^2}\right)^{-1}dy^2+y^2d\phi^2\,, \tag{2.2}$$

the geometry is uniquely specified by a single degree of freedom, the "radius" of the brane $A(\tau)$. The τ coordinate denotes proper time as measured along the brane-world. The computation of the GH boundary term has now reduced to that of computing the two non-trivial components of the second fundamental form. From eq. (2.2) we find (see appendix A for details):

$$\mathfrak{K}^\phi_\phi = \frac{1}{A}\left[1+\frac{A^2}{\ell^2}+\dot{A}^2\right]^{1/2}, \tag{2.3}$$

and

$$\mathfrak{K}^\tau_\tau = \left[\ddot{A}+\frac{A}{\ell^2}\right]\left[1+\frac{A^2}{\ell^2}+\dot{A}^2\right]^{-1/2}, \tag{2.4}$$

[1] In our convention, the extrinsic curvature is defined as $\mathfrak{K}_{\mu\nu} = 1/2(\nabla_\mu\hat{n}_\nu+\nabla_\nu\hat{n}_\mu)$, where $\hat{n}^\nu$ is the outward pointing normal vector to the boundary $\partial\Omega$. Lower Greek subscripts run from 0 to 2, capital Greek subscripts from 0 to d, and capital Latin subscripts from 0 to $(d-1)$. Throughout the paper we adopt geometrodynamic units so that $G\equiv 1$, $c\equiv 1$ and $\hbar\equiv L_p^2\equiv M_p^2$, where L_p and M_p are the Planck length and Planck mass, respectively.

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(where the dot denotes a derivative with respect to proper time). After integration by parts the gravitational lagrangian restricted to this minisuperspace may be identified as,

$$\mathcal{L} = \frac{L_p}{2}\left\{-\dot{A}\ \text{arcsinh}\left[\frac{\dot{A}}{\sqrt{1+A^2/\ell^2}}\right] + \sqrt{1+\frac{A^2}{\ell^2}+\dot{A}^2}\right\}. \tag{2.5}$$

The classical Wheeler-DeWitt hamiltonian is now easily extracted. In order to do this we compute the conjugate momentum to A,

$$p = \frac{\partial\mathcal{L}}{\partial\dot{A}} = -\frac{L_p}{2}\,\text{arcsinh}\left[\frac{\dot{A}}{\sqrt{1+A^2/\ell^2}}\right]. \tag{2.6}$$

This relation may be inverted to yield, $\dot{A} = -(1+A^2/\ell^2)^{1/2}\ \sinh(2p/L_p)$, so that the Wheeler-DeWitt hamiltonian is

$$\mathcal{H}_{\rm tot} \equiv p\dot{A} - \mathcal{L}_{\rm tot} = -2\pi AT - \frac{L_p}{2}\sqrt{1+\frac{A^2}{\ell^2}}\cosh\left(\frac{2p}{L_p}\right). \tag{2.7}$$

Equation (2.7) can be rewritten as,

$$\mathcal{H}_{\rm tot} = -2\pi AT - \frac{L_p}{2}\sqrt{1+\frac{A^2}{\ell^2}+\dot{A}^2}\,. \tag{2.8}$$

The hamiltonian constraint — which follows from the requirement of diffeomorphism invariance — is $\mathcal{H}_{\rm tot} = 0$, or equivalently,

$$\dot{A}^2 = -1 - A^2\left(\frac{1}{\ell^2} - 16\pi^2\frac{T^2}{L_p^2}\right). \tag{2.9}$$

Observe that the constraint equation is consistent with the covariant conservation of the stress-energy tensor and reproduces the classical Einstein field equations of motion. It is easy to see from eq. (2.9) that in order to obtain a real solution we need $T \neq 0$. Furthermore, the brane-world is (classically) bounded by a minimum radius

$$A_0^2 = \left(\frac{-1}{\ell^2} + \frac{16\pi^2T^2}{L_p^2}\right)^{-1}, \tag{2.10}$$

with $16\pi^2\ell^2T^2/L_p^2 > 1$. In other words, the brane bubbles appear classically at a critical size and then their expansion is governed by (2.9). Note that as the world approaches the minimum size the expansion tends to zero. Once the world is dynamically stable it experiences an everlasting expansion. However, we shall soon see that quantum effects permit well-behaved wave functions for vanishing T. With the classical dynamics of the model understood and the Wheeler-DeWitt hamiltonian at hand, quantization is straightforward. Canonical quantization proceeds via the usual

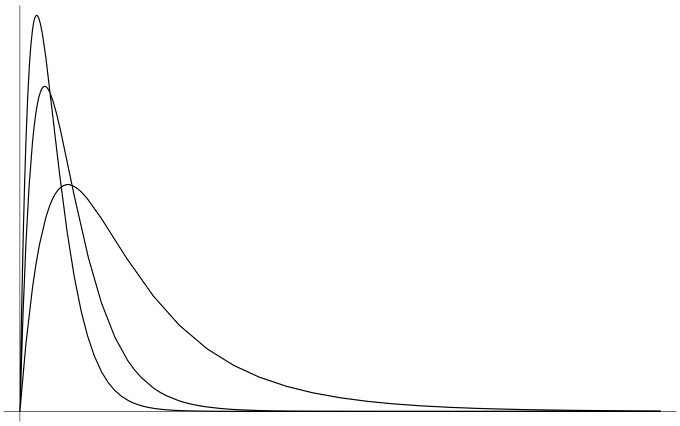

Figure 1: Some sample wavefunctions for $T = 0$ (upper is $\psi_{2,3}$, middle is $\psi_{1,2}$ and lower is $\psi_{0,1}$).

replacement $p \to -i\hbar\partial/\partial A$. Naturally, the resulting quantum hamiltonian has a factor order ambiguity. This factor-ordering ambiguity may be removed in a natural (though not unique) way by demanding that the quantum hamiltonian be hermitian,

$$\hat{\mathcal{H}}_{\rm tot} = \frac{L_p}{2}\left(1+\frac{A^2}{\ell^2}\right)^{1/4}\cos\left[2L_p\frac{\partial}{\partial A}\right]\left(1+\frac{A^2}{\ell^2}\right)^{1/4} + 2\pi AT\,. \tag{2.11}$$

That this hamiltonian is hermitian may formally be seen by Taylor-series expansion of the cosine. A more precise statement is that this hamiltonian acts on the Hilbert space of square-integrable functions defined on the half-interval $[0,\infty)$ subject to the constraint $\psi(0) = 0$. This is most easily seen by noting that the hamiltonian is hermitian on $L^2([0,\infty))$ only if $\psi(0) = 0$.[2]

The wave function of the brane-world is determined in the usual fashion by the Wheeler-DeWitt equation $\hat{\mathcal{H}}_{\rm tot}\psi(A) = 0$. For the special case of $T = 0$ we find the following solution:

$$\psi_{mn}(A) = C_{mn}(\varphi_m - \varphi_n)\,, \tag{2.12}$$

with

$$\varphi_j = \left(1+\frac{A^2}{\ell^2}\right)^{-1/4}\exp\left[-\left(j+\frac{1}{2}\right)\frac{\pi}{2}\frac{A}{L_p}\right]. \tag{2.13}$$

(See figure 1 for a plot of some of these wavefunctions). Here m and n are integer valued quantum numbers describing the internal state of the brane. Negatives values of m, n are not normalizable and so need to be discarded, as is the case when $m = n$. Note that the appropriate normalization is $\int|\psi|^2 dA = 1$, and that $\psi(0) = 0$, as required. In fact the two terms in ψ_{mn} individually satisfy the differential equation

[2]Here, the Hilbert space scalar-product is given by the sum over all possible configurations (i.e. sizes) of the brane-world. Henceforth, for any operator $\hat{\mathcal{O}}$ acting over any brane-wave-function ψ_j, $\langle\psi_k|\hat{\mathcal{O}}|\psi_j\rangle = \int_0^\infty \psi_k^*\,\hat{\mathcal{O}}\,\psi_j\,dA$. In other words, A is a parameter which governs the evolution of the brane along the null geodesic congruence of the extra dimension.

$\hat{\mathcal{H}}_{\rm gravity}\psi = 0$, but do not individually satisfy the boundary condition. By appropriate choice of C_{mn} these states may be normalized, though they are not orthogonal to one another. The normalization constant takes the rather complicated form:

$$\begin{aligned} C_{mn} = \Bigg\{ \frac{\ell\pi}{2} \Big[& H_0(\ell(m+1/2)\pi/L_p) + H_0(\ell(n+1/2)\pi/L_p) - \\ & - 2H_0(\ell(m+n+1)\pi/2L_p) - N_0(\ell(m+1/2)\pi/L_p) - \\ & - N_0(\ell(n+1/2)\pi/L_p) + 2N_0(\ell(m+n+1)\pi/2L_p) \Big] \Bigg\}^{-1/2}, \end{aligned} \tag{2.14}$$

where $H_0(z)$ is the Struve function and $N_0(z)$ is Neumann's function.[3] With the wavefunctions at hand, one can calculate the mean value of the "radius" of the brane, i.e.

$$\langle A \rangle = \frac{\int A|\psi_{mn}|^2 dA}{\int |\psi_{mn}|^2 dA}\,. \tag{2.15}$$

The integral in the numerator can be evaluated exactly but involves Meijer's G-function G^{31}_{13} and so the expression for $\langle A \rangle$ is not of much practical use (anyhow, by dimensional analysis one would expect that this number would be of order L_p). Using numerical integration we have found $\langle A \rangle_{0,1} = 0.54$, $\langle A \rangle_{1,2} = 0.01$ and $\langle A \rangle_{2,3} = 10^{-3}$ in units of L_p and for $\ell = 1$.

2.2 Qualitative behaviour of wavefunctions

In this subsection we take a first look at the problem of finding solutions to the Wheeler-DeWitt equation with $T \neq 0$ (in the next subsection we discuss the solutions in the WKB approximation). The relevant equation is:

$$\hat{\mathcal{H}}_{\rm tot}\psi(A) = 0 \tag{2.16}$$

or, more explicitly

$$\frac{L_p}{2}\left(1+\frac{A^2}{\ell^2}\right)^{1/4} \cos\left[2L_p\frac{\partial}{\partial A}\right]\left(1+\frac{A^2}{\ell^2}\right)^{1/4}\psi + 2\pi T A\psi = 0\,. \tag{2.17}$$

After defining $\varphi \equiv (1+A^2/\ell^2)^{1/4}\,\psi$, and the operator

$$\Delta \equiv \sum_{n=0}^{\infty}(-1)^n\frac{(2L_p)^{2n}}{2n!}\frac{\partial^{2n}}{\partial A^{2n}}\,, \tag{2.18}$$

we see that we have to solve

$$\Delta\varphi + \frac{4\pi T A}{L_p\sqrt{1+A^2/\ell^2}}\,\varphi = 0\,. \tag{2.19}$$

[3]These functions have the following respective integral representations: $H_0(z) = \frac{2}{\pi}\int_0^1 \frac{\sin(zt)\ dt}{\sqrt{1-t^2}}$ and $N_0(z) = -\frac{2}{\pi}\int_1^\infty \frac{\cos(zt)\ dt}{\sqrt{t^2-1}}$.

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In order to gain some intuition for the behaviour of the solutions of this equation we will look for solutions in the two limits $A/\ell \gg 1$ and $A/\ell \ll 1$. In the case $A/\ell \gg 1$, using a trial solution of the form $\varphi = e^{\lambda A}$, we find that eq. (2.19) reduces to the following condition:

$$\cos(2L_p\lambda) = -4\pi\frac{T\ell}{L_p}\,. \tag{2.20}$$

This is essentially the same condition as found before. Indeed, if $16\pi^2\ell^2T^2/L_p^2 > 1$ then λ will have pure imaginary values, leading to an oscillatory solution at infinity, that is not acceptable since it is not normalizable (it would in any case imply a delta function normalization, that we are not considering here) and should be something like a "classical" solution. On the other hand, if $16\pi^2\ell^2T^2/L_p^2 \leq 1$, λ will have two real solutions $\lambda_\pm$ of which one has to choose the negative one, with the same criteria of normalizability as before.

In order to analyze more carefully the behaviour of the wave function for $A/\ell \ll 1$, let us consider performing the following change of variables:

$$A \longrightarrow 2L_p a\,. \tag{2.21}$$

With this change of variables and after scaling $\psi \to \varphi$ as above, the Wheeler-DeWitt equation reads:

$$\tilde{\Delta}\varphi + \frac{8\pi T a}{\sqrt{1+4L_p^2a^2/\ell^2}}\varphi = 0\,, \tag{2.22}$$

where

$$\tilde{\Delta} = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\frac{\partial^{2n}}{\partial a^{2n}}\,. \tag{2.23}$$

It makes sense, since the factor L_p/ℓ is small, to analyze the behaviour of this equation for small values of the variable a and expand the square root in series.

The plot in figure 2 shows the behaviour, in the interval $[0,1]$, of the square of the wave function $|\psi|^2$ that solves numerically eq. (2.22), where we have considered eighteen orders of derivatives in $\tilde{\Delta}$. It is observed that $|\psi|^2$ has a maximum at $A \sim L_p$ as expected. It is worth to point out that the solutions with less derivatives have similar behaviour in the interval considered. It should be interesting to find a method to analyze the complete series.

Let us now consider a qualitative analysis of the possible wavefunctions. For this, let us set the AdS radius to one, and analyze the behaviour for different relations between T and L_p. In figure 3 we plot a schematic representation of the potential energy $V_{\text{eff}} = A^2(1/\ell^2 + 16\,\pi^2\,T^2/L_p^2)$. Classically, motion is confined to the region below the solid line (on which $V_{\text{eff}} = -1$). Strictly speaking, when $T = 2L_p$ classical motion is only allowed for $A > \alpha$, while for $T = L_p$ the condition is $A > \beta$. In this region the wave function ψ presents an oscillatory behaviour modulated by $(1+A^2/\ell^2)^{-1/4}$, whereas from the turning point to zero, ψ is exponentially decreasing.

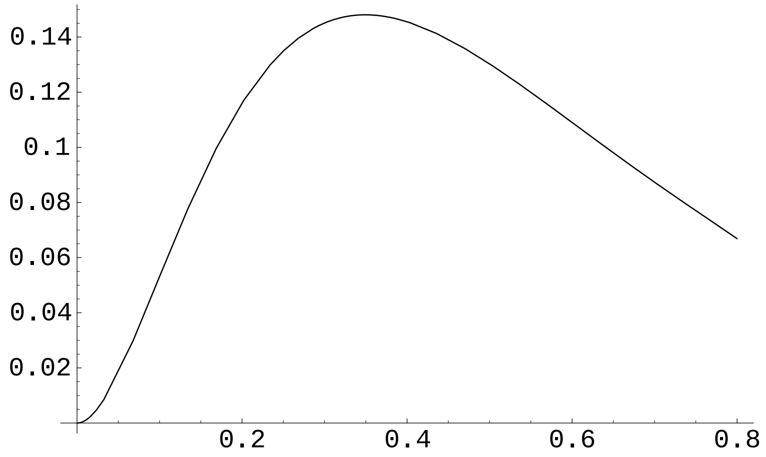

Figure 2: Square of the wave function ψ for small values of the variable a

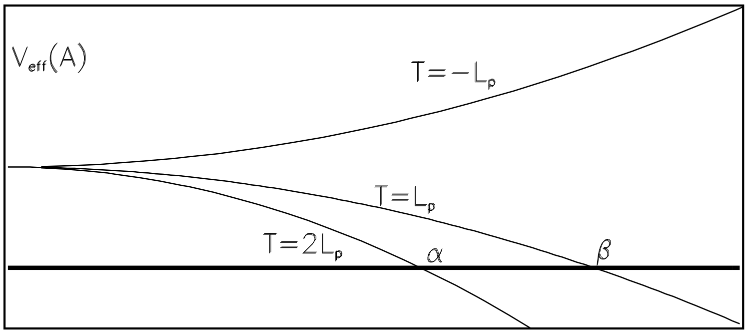

Figure 3: Effective potential $V_{\text{eff}}(A)$ as a function of A with the AdS radius $\ell = 1$. α and β are turning points.

The complete shape of ψ can be seen in figure 4. On the other hand, if $16\,\pi^2\ell^2T^2/L_p^2 < 1$, V_{eff} remains greater than -1 in the whole parameter space, and the classical motion is always forbidden. In this case, ψ can be expressed in terms of exponentials with real arguments, yielding just vacuum fluctuations. This is of course consistent with the behaviour we saw for $T = 0$.

2.3 WKB approximation

In this subsection we discuss solutions which are valid in the near-classical domain. Since the potential is slowly varying (see figure 3), one expects the wave function to closely approximate the free particle state wavefunction $\psi(A) = f(A)e^{i\,p\,A/\hbar}$. Thus, we will look for solutions of the form $\psi(A) = f(A)e^{iS(A)/\hbar}$. Following [16], the semi-classical quantization condition may be written in the generalized form,

$$\oint p(T, A)\, dA = (n_A + \delta)\hbar\,, \tag{2.24}$$

where n_A stands for the "radial" quantum number, and δ is related to the Maslov index [17]. For a hamiltonian quadratic in momenta, the usual WKB method shows

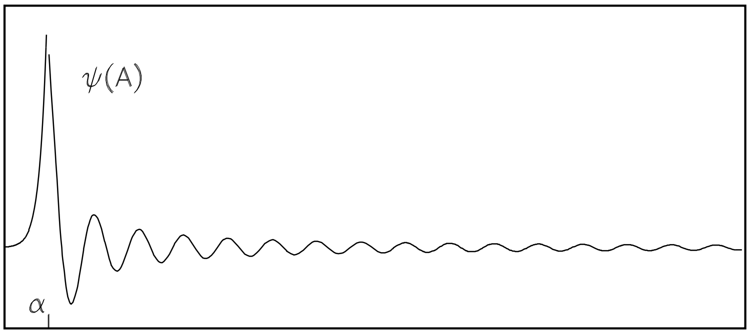

Figure 4: Qualitative behaviour of the wavefunction. The turning point is at $A = \alpha$.

that δ is typically a simple fraction. In other cases, δ depends on both the hamiltonian and the boundary conditions and is often transcendental. In the present discussion, a precise calculation of δ would add little to our understanding, thus, it will not be evaluated but shall merely be carried along as an arbitrary constant.

The precise form of the WKB wavefunction is determined by the following constraint. In the semi-classical limit ($\hbar \to 0$), the classical average in time of any quantity $Q(x)$,

$$\bar{Q}(x) = \frac{1}{\tau}\int_0^\tau Q(x(t))dt = \frac{1}{\tau}\int_0^\tau \frac{Q(x(t))}{v(x)}dx\,, \tag{2.25}$$

has to be equal to the quantum average,

$$\langle\psi|Q|\psi\rangle = \frac{\int |\psi(x)|^2 Q(x)dx}{\int |\psi(x)|^2 dx}\,; \tag{2.26}$$

where $v = \partial\mathcal{H}/\partial p$, and the classical time average $\tau = \int_0^\tau v^{-1}dx$. Thus, the semi-classical approximation in the classically allowed region is given by,

$$\psi_{\mathrm{WKB}}(A) = \left|\frac{\partial\mathcal{H}(p(T,A),A)}{\partial p}\right|^{-1/2} \exp\left[\pm\frac{i}{\hbar}\int^A p(T,x)dx\right], \tag{2.27}$$

while in the classical forbidden region it reads,

$$\psi_{\mathrm{WKB}}(A) = \left|\frac{\partial\mathcal{H}(p(T,A),A)}{\partial p}\right|^{-1/2} \exp\left[\pm\frac{1}{\hbar}\int^A p(T,x)dx\right]. \tag{2.28}$$

It is easily seen that for the typical hamiltonian quadratic in momentum this generalized prescription reduces to the usual WKB approximation.

The conjugate momenta results in a multi-valued function:

$$p(T,A) = \pm\frac{L_p}{2}\left\{\operatorname{arccosh}\left[\frac{-4\pi TA}{L_p\sqrt{1+A^2/\ell^2}}\right] + 2\pi i n\right\}. \tag{2.29}$$

Here $\operatorname{arccosh}(x)$ is taken to map $[1,\infty) \to [0,\infty)$, and $\pm$ refers to outgoing/ingoing directions. In this scheme the imaginary contribution to $p(T,A)$ does not contribute to the quantization condition. The quantum number n, however, does contribute when estimating the WKB wave function. In the classical allowed region we get,

$$\psi_{\rm WKB}(A) = \frac{\exp[-n\pi A/L_p]}{|-1 - A^2(16\pi^2T^2/L_p^2 - 1/\ell^2)|^{1/4}} e^{\pm i\Theta(A)}\,, \tag{2.30}$$

where

$$\Theta = \frac{1}{\hbar}\int^A \frac{L_p}{2}\,\mathrm{arccosh}\left[\frac{-4\pi T x}{L_p\sqrt{1+x^2/\ell^2}}\right] dx\,. \tag{2.31}$$

Note that in the limit $A/\ell \ll 1$,

$$\Theta = \frac{A}{2L_p}\,\mathrm{arccosh}\left[\frac{-4\pi T A}{L_p^2}\right]. \tag{2.32}$$

Thus, we recover the behaviour found in the previous subsection, ψ exponentially increases from zero to the turning point.

If we now flip $T \to -T$, and use $\mathrm{arccosh}(-x) = \mathrm{arccosh}(x) + i\pi$, we find that

$$\psi_{\rm WKB}(A) = \frac{\exp[-(n+1)\pi A/L_p]}{|-1 - A^2(16\pi^2T^2/L_p^2 - 1/\ell^2)|^{1/4}} e^{\pm\Theta(A)} \tag{2.33}$$

are WKB eigenmodes corresponding to an eigenvacuumenergy $-T$.

This semiclassical solution blows up at the turning points, where $\dot{A}$ goes to zero. This in itself may be tolerated if the wavefunction is normalizable. The matching of the wavefunction at the turning points may still be done by examining the wave equation more closely in the vicinity of the turning point.

3. Brane-world in AdS_{d+1}

3.1 Cosmology on the brane

We turn now to a more general analysis independent of the dimension, i.e. for AdS_{d+1} with $d > 1$. The expression for the total action is given by,

$$\begin{aligned} S_{\rm tot} = & \frac{L_p^{(3-d)}}{16\pi}\int_\Omega d^{d+1}x\sqrt{g}\left(R + \frac{d\,(d-1)}{\ell^2}\right) + \\ & + \frac{L_p^{(3-d)}}{8\pi}\int_{\partial\Omega} d^dx\sqrt{\gamma}\;\mathfrak{K} + T\int_{\partial\Omega} d^dx\sqrt{\gamma}\,. \end{aligned} \tag{3.1}$$

Let us also generalize the possible symmetries on the bulk which yield different Robertson-Walker like cosmologies. The most general AdS_{d+1} metric can be written as,

$$ds^2 = -\left(k + \frac{y^2}{\ell^2}\right)dt^2 + \left(k + \frac{y^2}{\ell^2}\right)^{-1} dy^2 + y^2 d\Sigma_k^2\,, \tag{3.2}$$

where k takes the values $0, -1, 1$ for flat, hyperbolic, or spherical geometries, respectively and where $d\Sigma_k^2$ is the corresponding metric on the unit $(d-1)$-dimensional plane, hyperboloid, or sphere. It should be stressed that if $k=-1$, an event horizon appears at $y=\ell$. With this in mind, one can trivially generalize the discussion in the appendix to get,

$$\mathfrak{K}^{\phi_i}_{\phi_i} = \frac{1}{A}\left[k + \frac{A^2}{\ell^2} + \dot{A}^2\right]^{1/2}, \tag{3.3}$$

and

$$\mathfrak{K}^{\tau}_{\tau} = \left[\ddot{A} + \frac{A}{\ell^2}\right]\left[k + \frac{A^2}{\ell^2} + \dot{A}^2\right]^{-1/2}, \tag{3.4}$$

where i runs from 1 to $(d-1)$. In terms of these quantities, the Einstein equation reads [18],

$$T g_{\Xi\Upsilon}\delta^{\Xi}_{A}\,\delta^{\Upsilon}_{B} = \frac{L_p^{3-d}}{4\,\pi}\left[\mathfrak{K}_{AB} - \mathrm{tr}(\mathfrak{K})g_{\Xi\Upsilon}\delta^{\Xi}_{A}\,\delta^{\Upsilon}_{B}\right]. \tag{3.5}$$

Its non-trivial components are,

$$T = -\frac{L_p^{(3-d)}}{4\pi}\frac{(d-1)}{A}\left(k + \frac{A^2}{\ell^2} + \dot{A}^2\right)^{1/2}, \tag{3.6}$$

and

$$T = -\frac{L_p^{(3-d)}}{4\pi}\left\{\frac{(d-2)}{A}\left(k + \frac{A^2}{\ell^2} + \dot{A}^2\right)^{1/2} + \frac{\ddot{A} + A/\ell^2}{\sqrt{k + \dot{A}^2 + A^2/\ell^2}}\right\}. \tag{3.7}$$

It is easily seen that eqs. (3.6) and (3.7) imply the conservation of the stress energy. The evolution of the system is thus governed by,

$$\dot{A}^2 = -k - A^2\left(\frac{1}{\ell^2} - \frac{16\pi^2\,T^2}{(d-1)^2\;L_p^{2(3-d)}}\right). \tag{3.8}$$

Recall that in the spherical case, the classical behaviour of the brane is bounded by a minimum radius

$$A_0^2 = \left(-\frac{1}{\ell^2} + \frac{16\,\pi^2\,T^2}{(d-1)^2\;L_p^{2(3-d)}}\right)^{-1}, \tag{3.9}$$

but once the brane reaches that "size" it expands forever. On the other hand, if $k=-1$ (hyperbolic symmetry) and

$$\frac{16\pi^2 T^2\ell^2}{(d-1)^2 L_p^{2(3-d)}} \geq 1\,, \tag{3.10}$$

the classical solution does not have turning points; whereas if

$$\frac{16\pi^2 T^2\ell^2}{(d-1)^2 L_p^{2(3-d)}} < 1\,, \tag{3.11}$$

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the classical solution has two turning points. It should be remarked, however, that if $k = -1$, the brane crosses an event horizon (at $A = \ell$) in a finite proper time. Thus, if $A < \ell$ the brane is "localized" in time and not in the bulk. If $k = 0$, one obtains a solution only if the inequality (3.10) is satisfied. In the critical case the solution represents the RS_{d+1} brane-world. At this stage, it is noteworthy that a comprehensive analysis of a domain wall that inflates, either moving through the bulk or with the bulk inflating too, was first discussed by Chamblin-Reall [11].

3.2 Semiclassical corrections

With the field equations for an expanding $(d-1)$-brane in hand, the generalization of the WKB approximation to AdS_{d+1} is straightforward. Of particular interest is AdS_5.[4] Let us specialize again to the case of a spherically symmetric brane. In such a case, eq. (3.1) can be re-written as

$$S_{\rm tot} = \frac{1}{L_p}\int d\tau \left\{ -\frac{A^3}{3\ell^2}\frac{\sqrt{\dot{A}^2 + A^2/\ell^2 + 1}}{1 + A^2/\ell^2} + 3A\sqrt{1 + A^2/\ell^2 + \dot{A}^2} - \right.$$
$$\left. - 2A\dot{A}\,\mathrm{arcsinh}\left[\frac{\dot{A}}{\sqrt{1 + A^2/\ell^2}}\right]\right\} + T\int_{\partial\Omega} d^4x\sqrt{\gamma}\,. \qquad (3.12)$$

For positive eigenvalues of T, the solution in the classical allowed region is then given by,

$$\psi_{\rm WKB}(A) = \frac{\exp[-2\,\pi\, n\,(A/L_p)^2]}{|-1 - A^2/\ell^2 + G^2)|^{1/4}}\, e^{\pm i\int^A p\,dx}\,, \qquad (3.13)$$

with $p \equiv \partial\mathcal{L}_5/\partial\dot{A}$, and $G(A) = 4\pi A^2 T L_p/3$. The oscillating part will be a real exponential term in the classically forbidden region.

4. Relation to AdS/CFT correspondence

Another, seemingly different, but in fact closely related subject we will discuss in this section is the AdS/CFT correspondence [19]. This map provides a "holographic" projection of the AdS gravitational system into the physics of the gauge theory. In the standard non-compact AdS/CFT set up, gravity is decoupled from the dual boundary theory. However, it is natural to suppose that any RS-like model should properly be viewed as a coupling of gravity to whatever strongly coupled conformal theory the AdS geometry is dual to. In the following discussion, inspired in [20], we unfold on this hypothesis: the most general action for a RS-like model in AdS_{d+1} is given by

$$S_{\rm RS} = S_{\rm EH} + S_{\rm GH} + 2S_1 + S_m\,, \qquad (4.1)$$

[4] Note that if $k = 0$ and $T = 3/(4\pi L_p\ell)$, one recovers the RS-world.

where S_1 is the counterterm $(T/2)\int d^dx\sqrt{\gamma}$. The last term S_m is the action for matter on the brane which was not included in eq. (3.1), but it is included here for completeness. Now, to apply the AdS/CFT-correspondence, there is the question of the definition of the gravitational action in AdS_{d+1}. The standard action — corresponding to the two first terms of eq. (3.1) — is divergent for generic geometries and one must add certain "counterterms" to obtain a finite action [21, 22]. Then we have schematically, $S_{\rm grav} = S_{\rm EH} + S_{\rm GH} + S_1 + S_2 + S_3 + \cdots$, where S_k is of order $2(k-1)$ in derivatives of the boundary metric. Specifically, S_2 and S_3 are the counterterms discussed in [20]. They are expressed in terms of the boundary metric: $S_2 \propto \int d^dx\sqrt{\gamma}\tilde{R}$, and $S_3 \propto \int d^dx\sqrt{\gamma}(\tilde{R}_{ij}\tilde{R}^{ij} - \frac{d}{4(d-1)}\tilde{R}^2)$. Some of the higher-order counterterms were computed in [22]. For a given dimension d, however, one only needs to add a *finite* number of counterterms, specifically terms of order $2n < d$ in derivatives of the boundary metric.

In order to apply the AdS/CFT-correspondence we should require that the symmetries on both sides of the correspondence match. The Weyl anomaly was computed in [23] for gravity theories in AdS_{d+1} and we can then apply this result to fix the possible counterterms. For d odd there is no such anomaly and the divergent part of the (super)gravity action is canceled by the addition of the above mentioned counterterms. This implies, for example, that for AdS_4 we should only add S_1 and S_2. For d even there is a non-vanishing anomaly [23]. For AdS_3 this means that both S_1 and S_2 should be added and for AdS_5 we should add the terms S_1, S_2 and S_3. So, the requirement of finiteness of the action *together* with the matching of Weyl anomalies fixes the precise form of the supergravity action in AdS_{d+1}.

Now, using the AdS/CFT-correspondence, one can easily show that the RS-model in dimension $d+1$ is dual to a d-dimensional CFT (which we call the RS CFT) with a coupling to matter fields and the domain wall given by the action $2S_2 + 2S_3 + \cdots + S_m$, where we should remember that for AdS_3 and AdS_4, the S_3-term is absent but appears in all higher-dimensional cases. To illustrate this point, let us now analyze the AdS/CFT for the simplest three-dimensional example. We will work in euclidean space in order to avoid definition problems in the path integral. In this case the RS action (without matter) is given by

$$S_{\rm RS} = -\frac{L_p}{16\pi}\int_\Omega d^3x\sqrt{g}\left(R + \frac{2}{\ell^2}\right) - \frac{L_p}{8\pi}\int_{\partial\Omega} d^2x\sqrt{\gamma}\,\mathfrak{K} - \frac{L_p}{4\pi}\int_{\partial\Omega} d^2x\sqrt{\gamma}\,, \qquad (4.2)$$

which is essentially the same as in eq. (2.1) but now with the tension T fixed to be $L_p/(4\pi)$. (More on this below.) Our set-up consists of two regions R_1 and R_2 bounded by a two-dimensional domain wall and on each of these regions the metric is AdS_3 $g_{\mu\nu}$ which induces the metric γ_{ij} on the wall. Following [20, 24], let us compute the partition function obtained by integrating over the bulk metric with boundary value γ_{ij} on the wall:

$$Z_{\rm RS}[\gamma] = e^{-2S_1} \left(\int_{R_1 \cup R_2} \mathcal{D}g \; e^{-S_{\rm EH}[g] - S_{\rm GH}[g]} \right), \tag{4.3}$$

where the integral is over the two patches R_1 and R_2 of AdS. (Note that even though $S_{\rm GH}$ is a two-dimensional term it depends on the bulk metric through the extrinsic curvature of the domain wall and can therefore not be taken out of the path integral.) Since the integral over the two regions of AdS-space are independent, we can write it as an integral over a single patch of AdS-space:[5]

$$Z_{\rm RS}[\gamma] = e^{-2S_1} \left(\int_{R_1} \mathcal{D}g \; e^{-S_{\rm EH}[g] - S_{\rm GH}[g]} \right)^2. \tag{4.4}$$

Now according to the discussion above, the partition function for a consistent gravity theory in AdS_3, with finite mass of spacetime and appropriate central charge, is

$$\begin{aligned} Z_{\rm grav}[\gamma] &= \int_{[\gamma]} \mathcal{D}g \; e^{-S_{\rm EH}[g] - S_{\rm GH}[g] - S_1[\gamma] - S_2[\gamma]} \\ &= e^{-S_1[\gamma] - S_2[\gamma]} \int_{[\gamma]} \mathcal{D}g \; e^{-S_{\rm EH}[g] - S_{\rm GH}[g]} \\ &= e^{-W_{\rm CFT}[\gamma]}, \end{aligned} \tag{4.5}$$

and according to the AdS/CFT it should be identified with the generating functional for connected Green's functions of the RS CFT as above. By combining eq. (4.4) and (4.5) we finally obtain:

$$Z_{\rm RS}[\gamma] = e^{-2W_{\rm CFT}[\gamma] + 2S_2[\gamma]}. \tag{4.6}$$

This shows that the RS-like model in AdS_3 is equivalent to a CFT coupled to gravity with action $2S_2$. This dual gravity theory is actually two dimensional since $2S_2$ is the Einstein-Hilbert action for two-dimensional gravity. Similar correspondences can be derived in higher-dimensional cases [20]. This alone, however, does not tell us what the RS CFT actually is,[6] but rather that the RS model in $d+1$ dimensions can be viewed as a d-dimensional gravity (including corrections) coupled to a CFT with matter.[7] And so, for example, in the case of AdS_5 this is another way to see why gravity is trapped on the four-dimensional domain wall and why there are corrections to Einstein gravity.

[5] Note that the result of the integral over the regions $R_1 \cup R_2$ is not the addition of the integrals, but the product. Indeed, since we are dealing with independent processes, we have the product of the probabilities amplitudes instead of the sum, that would produce 'interference effects' not present in the RS set-up.

[6] The boundary CFT can be found for the case of AdS_3 [25].

[7] Related ideas were discussed in [26].

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Up to this point we have kept the tension of the domain wall, T, arbitrary. Because of the various bounds described in sections 2 and 3 for different behaviours of the braneworld, it is important to see what one might expect. It is well known that gravity in asymptotically AdS_3 spacetime has a holographic description as a $(1+1)$-dimensional conformal field theory with central charge $c = 3\,\ell\, M_p/2$ [27]. In order to recover the geometry discussed in section 2, one must glue two copies of such bounded AdS_3 spacetimes, and then integrate over boundary metrics. Consequently, one has two copies of the matter action on the boundary, with total central charge $c = 3\,\ell\, M_p$. In addition, if $\tilde{R} > 0$ the conformal anomaly of the CFT increases the effective tension on the domain wall, $T > L_p/4\pi\ell$, yielding a de Sitter universe with an effective cosmological constant driving inflation.[8] An (early) inflationary epoch looks very promising. The tremendous expansion during inflation may blow up a small sized region of the world (which was causally connected before inflation) to a size much greater than our current horizon. Therefore, it can be expected that the observable part of the brane looks smooth and flat, regardless of the initial curvature of the brane that inflated.[9] Furthermore, if we consider conformal matter on the brane the inflationary phase is unstable and could decay into a matter dominated universe with thermalized regions, in agreement with current observations [29]. Another interesting process which could lead to brane-world reheating is as follows: during inflation trapped regions of false vacuum (within their Schwarzchild radii) caught between bubbles of true vacuum may give rise to the creation of primordial black strings. Now, it is well known that the black string solution suffers from a Gregory-Laflamme instability [30] leading to the formation of stable black cigars on the brane. In addition, it was shown in [31] that the nucleation of supermassive bulk black holes is highly supressed compared to the above mentioned process. Thus, prompted by the conventional arena [32], one could speculate that the Hawking-evaporation of primordial black cigars slows down inflation. On the other hand, one could assume the existence of such a bulk black hole. Even in this case, the (brane-world/bulk-black-hole) system evolves towards a configuration of thermal equilibrium as was recently shown in [33].

Despite the fact that it is contrary to the spirit of RS-worlds, it would be nice to add "matter fields" in the bulk to study the quantum cosmology and the dual CFT coupled to gravity that in this case should be deformed by the insertion of

[8]A few words of caution; it is quite possible that the truncation from an infinite number of degrees of freedom down to only one degree of freedom, $A(\tau)$, has also drastically truncated the real physics. Unfortunately, a treatment using Wheeler's full superspace is beyond the scope of our present calculation abilities.

[9]Note that a flat Robertson-Walker Universe requires a total energy density equal to the critical density $\rho_{\rm cr}$, whereas ordinary matter contributes only about a 5% of $\rho_{\rm cr}$. A novel solution to this problem consistent with a large body of observations is the so-called "Manifold Universe" [28].

operators. Even though many kind of interesting phenomena are recognized, brane-world cosmology remains thoroughly non understood. The lower-dimensional model here discussed can hopefully illuminate the "physical AdS_5 cosmology".

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A. Surgery on the brane

Here we present a calculation of the second fundamental form of the metric in eq. (2.2) (it should be remarked that this calculation is a direct analog to that of ref. [34], and it is included just for the sake of completeness).

Let us start by introducing a Gaussian normal coordinate system in the neighborhood of the brane. We shall denote the one-dimensional surface swept out by the brane by Σ. Let us introduce a coordinate system $\phi_\perp$ on Σ. Next we consider all the geodesics which are orthogonal to Σ, and choose a neighborhood N around Σ so that any point $p \in N$ lies on one, and only one, geodesic. The first coordinate of p is determined by the intersection of this geodesic with Σ. The full set of spatial coordinates is then given by $(\phi_\perp; \eta)$, while the surface Σ under consideration is taken to be located at $\eta = 0$ so that eq. (2.2) can be rewritten as

$$ds^2 = -\left(1 + \frac{y^2}{\ell^2}\right) dt^2 + d\eta^2 + y^2 d\phi^2 \,, \tag{A.1}$$

fixed by the relation, $dy/d\eta = (1 + y^2/\ell^2)^{1/2}$. The second fundamental form in such a coordinate-system reads

$$\mathfrak{K}_{\mu\nu} \equiv \frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial \eta}\bigg|_{\eta=0,y=A} \,, \tag{A.2}$$

and its non-trivial components are

$$\begin{aligned} \mathfrak{K}^t_t &= \frac{A}{\ell^2}\left(1 + \frac{A^2}{\ell^2}\right)^{-1/2} \,, \\ \mathfrak{K}^\phi_\phi &= \frac{1}{A}\left(1 + \frac{A^2}{\ell^2}\right)^{1/2} \,. \end{aligned} \tag{A.3}$$

To analyze the dynamics of the system, we permit the radius of the brane to become a function of time $A \to A(\tau)$. Recall that the symbol τ is used to denote proper time as measured by co-moving observers on the brane-world. Let the position of the brane be described by $x^\mu(\tau,\phi) \equiv (t(\tau), A(\tau), \phi)$, so that the velocity of a piece of stress-energy at the brane ($u^\mu u_\mu = -1$) is

$$u^\mu \equiv \frac{dx^\mu}{d\tau} = \left(\frac{dt}{d\tau}, \frac{dA}{d\tau}, 0\right). \tag{A.4}$$

We remind the reader that

$$ds^2 = -\left(1+\frac{A^2}{\ell^2}\right)dt^2 + \left(\frac{dA}{dt}\right)^2\left(1+\frac{A^2}{\ell^2}\right)^{-1}dt^2 + A^2 d\phi^2 \tag{A.5}$$

so,

$$d\tau^2 = -dt^2\left\{-\left(1+\frac{A^2}{\ell^2}\right) + \left(\frac{dA}{dt}\right)^2\left(1+\frac{A^2}{\ell^2}\right)^{-1}\right\} \tag{A.6}$$

or equivalently,

$$d\tau^2 = -dt^2\left\{-\left(1+\frac{A^2}{\ell^2}\right)^2 + \left(\frac{dA}{dt}\right)^2\right\}\left(1+\frac{A^2}{\ell^2}\right)^{-1}. \tag{A.7}$$

Since

$$\frac{dA}{dt} = \frac{dA}{d\tau}\frac{d\tau}{dt}, \tag{A.8}$$

we first get,

$$-\left(1+\frac{A^2}{\ell^2}\right)\left(\frac{d\tau}{dt}\right)^2 = -\left(1+\frac{A^2}{\ell^2}\right)^2 + \dot{A}^2\left(\frac{d\tau}{dt}\right)^2 \tag{A.9}$$

and then,

$$\frac{dt}{d\tau} = \frac{\sqrt{\dot{A}^2 + A^2/\ell^2 + 1}}{1 + A^2/\ell^2}. \tag{A.10}$$

Let us denote by $\hat{n}^\mu$ the unit normal vector to the brane, which satisfies $u^\mu \hat{n}_\mu = 0$ and $\hat{n}^\mu \hat{n}_\mu = 1$; its components are $\hat{n}^\mu = (\dot{A}/(1+A^2/\ell^2), (1+A^2/\ell^2+\dot{A}^2)^{1/2}, 0)$, such that the coordinate y is increasing in the direction $\hat{n}^\mu$. Thus we obtain

$$\mathfrak{K}^\phi_\phi = \frac{1}{y}\frac{\partial y}{\partial \eta}\bigg|_{y=A} = \frac{1}{A}\left(1+\frac{A^2}{\ell^2}+\dot{A}^2\right)^{1/2}. \tag{A.11}$$

To evaluate $\mathfrak{K}^\tau_\tau$ one can proceed in two alternative ways. First one can simply use the definition $\mathfrak{K}_{\mu\nu} = \frac{1}{2}\nabla_{(\mu}\hat{n}_{\nu)}$, giving:

$$\mathfrak{K}_{tt} = \frac{1}{2}\nabla_{(t}\hat{n}_{t)} = \frac{d\hat{n}_t}{d\tau}\frac{d\tau}{dt} - \Gamma^\eta_{tt}\,\hat{n}_\eta = -\frac{1+A^2/\ell^2}{\sqrt{1+\dot{A}^2+A^2/\ell^2}}\left(\ddot{A}+\frac{A}{\ell^2}\right), \tag{A.12}$$

that using

$$\mathfrak{K}_{\tau\tau} = \frac{\partial x^\mu}{\partial x^\tau}\frac{\partial x^\nu}{\partial x^\tau}\mathfrak{K}_{\mu\nu}, \tag{A.13}$$

immediately yields

$$\mathfrak{K}^{\tau}_{\tau} = \mathfrak{K}^{t}_{t} = \frac{\ddot{A} + A/\ell^2}{\sqrt{1 + \dot{A}^2 + A^2/\ell^2}} \,. \tag{A.14}$$

Alternatively, one can easily check this last result by observing that

$$\begin{aligned} \mathfrak{K}^{\tau}_{\tau} \equiv -\mathfrak{K}_{\tau\tau} &= -u^\mu u^\nu \mathfrak{K}_{\mu\nu} = -u^\mu u^\nu \nabla_\mu \hat{n}_\nu = u^\mu \hat{n}_\nu \nabla_\mu u^\nu \\ &= \hat{n}_\mu (u^\nu \nabla_\nu u^\mu) = \hat{n}_\mu q^\mu \,, \end{aligned} \tag{A.15}$$

where q^μ is the four acceleration of the brane. Now, by the spherical symmetry of the problem the four acceleration is proportional to the unit normal, $q^\mu \equiv q\,\hat{n}^\mu$, so $\mathfrak{K}^{\tau}_{\tau} = q$. To explicitly evaluate the four acceleration, utilize the fact that $\xi^\mu \equiv \partial^\mu_t \equiv (1,0,0)$ is a Killing vector for the underlying geometry. At the brane, the components of this vector are $\xi_\mu = (-[1 + A^2/\ell^2], 0, 0)$, so that $\xi_\mu \hat{n}^\mu = -\dot{A}$ and $\xi_\mu u^\mu = -(1 + A^2/\ell^2 + \dot{A}^2)^{1/2}$. With this in mind, comparing

$$\frac{d}{d\tau}(\xi_\mu u^\mu) = \xi_\mu\, q\, \hat{n}^\mu = -q\,\dot{A} \,, \tag{A.16}$$

and

$$\frac{d}{d\tau}(\xi_\mu u^\mu) = -\dot{A} \frac{A/\ell^2 + \ddot{A}}{\sqrt{1 + A^2/\ell^2 + \dot{A}}} \,, \tag{A.17}$$

we get

$$\mathfrak{K}^{\tau}_{\tau} = \frac{A/\ell^2 + \ddot{A}}{\sqrt{1 + A^2/\ell^2 + \dot{A}}} = \frac{d}{d\tau} \left\{ \operatorname{arcsinh} \left[\frac{\dot{A}}{\sqrt{1 + A^2/\ell^2}} \right] \right\} + \frac{A}{\ell^2} \frac{dt}{d\tau} \,; \tag{A.18}$$

this result agrees with that of eq. (A.14). Having calculated the non-trivial components of the second fundamental form we can now derive a simpler expression for the relevant gravity-action (2.1) in AdS_3. Since $\sqrt{g}\, d^3x \to 2\pi\, A\, dA\, dt$ and $\sqrt{\gamma}\, d^2x \to 2\pi\, A\, d\tau$ an integration by parts finally leads to

$$S_{\text{gravity}} = \frac{L_p}{2} \int d\tau \left\{ -\dot{A}\ \operatorname{arcsinh} \left[\frac{\dot{A}}{\sqrt{1 + A^2/\ell^2}} \right] + \sqrt{1 + \frac{A^2}{\ell^2} + \dot{A}^2} \right\} . \tag{A.19}$$

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